Occam's razor vs. magnetic field

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One should not increase, beyond what is necessary,
the number of entities required to explain anything.

William Occam ~1300

Abstract. The concept of magnetic field leads to many paradoxes. Abandoning this notion and using Weber's formula for the elementary force of interaction between moving charges, it is possible to deduce Faraday's law of electromagnetic induction, Ampere's force law for currents interaction etc. As the result the paradoxes are resolved by themselves. Moreover, Weber's formula for calculation of the force exerted on the electron moving between the plates of a parallel-plate capacitor does not require the electron mass to vary with its velocity.

1. Paradoxes of the magnetic field

In physics the Lorentz force is the combination of electric and magnetic force on a point charge due to electromagnetic fields. If a particle of charge q moves with velocity v in the presence of an electric field E and a magnetic field B, then it will experience a force (in SI units)

$$\vec{F}_L = q\vec{E} + q\,\vec{v} \times \vec{B} \qquad (1)$$

However the charge velocity depends on the reference frame; consequently the Lorentz magnetic force F_{mag} = q **v**×**B** is different for different reference frames. This is a guaranteed factor leading to possible paradoxes. One such paradox was recently published by M. Mansuripur [1].

Assume the test charge is at rest near a pole of a magnet (Fig.1a). The magnet is neutral, so no electric force is exerted on the charge. The charge is in the magnetic field but v = 0, hence F_{mag} = 0. As a result, no force is exerted on the charge. But in the frame moving to the left, the charge has non-zero velocity (Fig.2b), therefore the charge experiences a non-zero magnetic force which contradicts our previous conclusion.

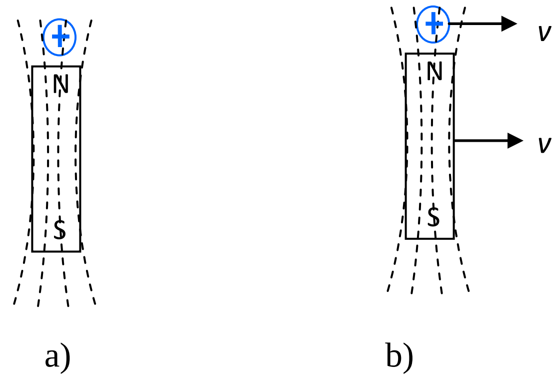

Fig.1. Absence (a) or presence (b) of magnetic force depends on the choice of reference frame (Mansuripur's paradox [1])

To avoid the conflict, one has to decide that for the Lorentz force calculation the charge velocity should be taken relative to the object creating the magnetic field. We will see below that this assertion conflicts with Faraday's paradox.

The moving magnet and conductor problem [2] is a famous thought experiment described by A. Einstein in his 1905 paper "On the electrodynamics of moving bodies."

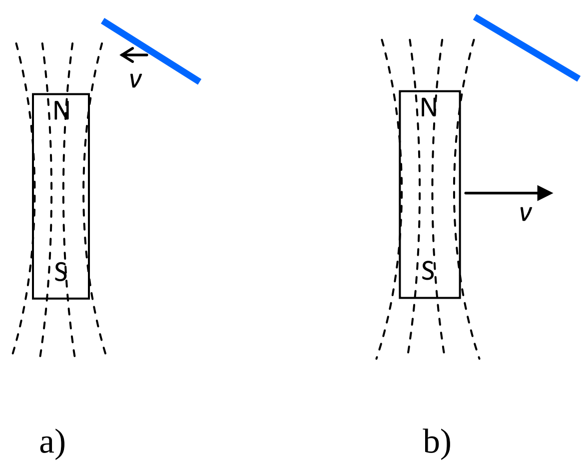

Fig.2. Moving magnet and conductor problem: in the magnet's reference frame (a) a magnetic force is exerted upon electrons of the conductor, while in the reference frame of the conductor (b) the same force becomes electric.

If a conductor moves in the magnetic field of the magnet (Fig.2a), the conductor charges experience a Lorentz magnetic force in the magnet reference frame. On the other hand, for an observer moving together with the conductor (Fig.2b), the latter is at rest (v=0) and there is no Lorentz force. Since the existence of the force is not relative, some other force acts on the electrons of the conductor instead; this force is called the electromotive force of induction (emf). Thus the difference between the electric and magnetic forces is subjective. We may assume that one of these two fields is artificial and has no relation to reality.

Wikipedia [3] came to the same conclusion: “The Lorentz transformation of the electric field of a moving charge into a non-moving observer's reference frame results in the appearance of a mathematical term commonly called the magnetic field.”

The famous Faraday paradox [4] has led to a debate as to whether a magnetic field rotates with a magnet or not. Michael Faraday discovered this in the 1820’s and so far it has no satisfactory explanation within the standard paradigm. A conducting disc rotates near a pole of a cylindrical magnet (Fig.3). A voltmeter is connected to the axle of the disc and to its rim via sliding contacts. The voltmeter shows the presence of a voltage that can be explained by the Lorentz force acting on the mobile electrons in the magnetic field.

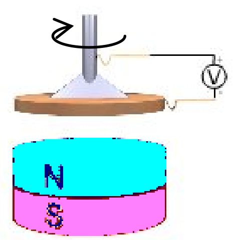

Fig.3. Faraday homopolar generator.

Paradoxically, when the cylindrical magnet in Fig.3 rotates together with the disk, the voltmeter registers a voltage as well. According to the conclusion reached above after the first paradox, if the disc and the magnet are stationary relative to each other, the velocities of the disc electrons with respect to the magnet are zero and the emf should not be produced. Even more paradoxical is the case when the disc is held stationary while the magnet is spun on its axis but an emf is not produced (see video in [5]). That is, relative motion of the disk and the magnet in some cases produces an emf but in other cases does not.
We will explain this paradox from the standpoint of an alternative approach in Section 4.

2. The alternative hypothesis

Carl Friedrich Gauss was the first to propose the alternative hypothesis which abandons the concept of the magnetic field [6]. In 1835 Gauss sent a letter to his experimental collaborator Wilhelm Weber in which he suggested an elementary force between two moving electric charges q_1 and q_2 (in SI units):

$$F_G = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\left[1 + \frac{u^2}{c^2} - \frac{3}{2}\frac{\dot{r}^2}{c^2}\right]$$

where r is the distance between the charges, $\dot{r} = \frac{dr}{dt}$, $\vec{r}$ is the vector connecting the two charges, $\vec{u} = \frac{d\vec{r}}{dt}$ is their relative velocity, and ε_0 is the vacuum permittivity. The force is directed along the line joining the charges. According to this hypothesis the interaction between two charges does not depend only on the distance between them, but also on their relative velocity, so that a uniform motion of the observer can not affect the force between the charges.

In 1846 Weber [7] published an improved formula for the same force which includes in addition a relative acceleration of the charges $\vec{a}$ (see also [8]). According to Weber's formula a charge q_1 exerts upon a charge q_2 (and vice versa) the force

$$F_W = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\left[1 + \frac{u^2}{c^2} - \frac{3}{2}\frac{\dot{r}^2}{c^2} + \frac{\vec{r}\cdot\vec{a}}{c^2}\right]. \qquad (2)$$

Equation (2), one of several possible forms of Weber's formula, is selected to show clearly its difference from that of Gauss. Coulomb's law [9] is a special case of the force when the charges are at relative rest. Weber chose this formula since it provides the correct expression for Ampère's force for interaction of currents. Walter Ritz [10] gave physical substantiation to Weber's formula: in the case of moving charges there is a delay of their electric fields; as a result the force of interaction depends on v^2/c^2. This term is absent in Lorentz's formula (1). We will see in section 5 that Lorentz introduced this term in mass variation.

In his 1846 paper Weber showed application of his formula to the induction phenomenon and its complete compatibility with Ampere's law. Based on this formula many authors derived the electromotive force of induction without the Lorentz force and without the notion of magnetic field (see [11]). Helmholtz objected to Weber's formula from energy considerations, but Weber showed that Helmholtz's assertion was incorrect (see Maxwell [12], p484).

V. Bush [13] perfectly described the gist of the problem: "Historically, it is quite evident why classical theory was built up in its present form, for electrostatic and magnetic effects were independently known before their inter-relationships were examined, and were considered entirely separate entities."

Differences between the magnetic and electric phenomena are only apparent. Magnets are usually neutral, therefore the electrostatic forces cancel each other out, and only an additional "magnetic force" manifests itself by its dependence on the relative velocities of the charges. In simple cases the Lorentz force coincides with Weber's force, which helped with the calculation of electric motors, generators, transformers and so on. But in case of an arbitrarily moving magnet (as in the Faraday paradox) there is no clear explanation as to whether the magnetic field moves with it or not. In the approach based on Weber's law, the magnetic field does not exist, therefore the question of its movement does not arise [13].

In 2001 L.Hecht wrote [14]: "Weber's discovery made a revolution in physics, the full implications of which are still unrealized".

3. Deduction of Ampère's force law from Weber's formula

The need for the deduction is due to the fact that in Weber's time it was not yet known that the positive charges are fixed in metals, so Weber deduced Ampere's law assuming Fechner's hypothesis: an electric current is a flow of positive and negative charges. One can consider Weber's deduction as based on a false assumption, but there is another approach, namely, as if Weber considered the interaction between two parallel wires moving along their length at a constant speed. Then the positive charges have non-zero velocities and the question is whether such movement has an effect on Ampère's force [15]. If not, then Weber's assumption does not change the validity of his deduction.

Our plan consists in deducing the force of interaction between two moving parallel currents based on Weber's formula (2) and comparing it to Ampère's force for stationary conductors carrying currents.

We discuss two horizontal wires carrying currents I_1 and I_2 (see Fig.4), which move along their length so that their positive charges have constant speeds v_1^+ and v_2^+ accordingly. Let v_1^- and v_2^- denote velocities of the negative charges of the two wires. Using the known relation v dq = I ds, where ds is a segment of wire, dq is the charge on ds, we obtain for the first wire

$$I_1 ds_1 = v_1^+ dq_1^+ + v_1^- dq_1^- = (v_1^+ - v_1^-)|dq_1| \quad (3)$$

where the neutrality of the wire was used: $dq_1^+ = -dq_1^-$.

A similar formula holds for the second wire:

$$I_2 ds_2 = (v_2^+ - v_2^-)|dq_2| \quad (3')$$

Let a be the distance between the parallel direct currents I_1 and I_2, and r be the distance between the segments ds_1 and ds_2.

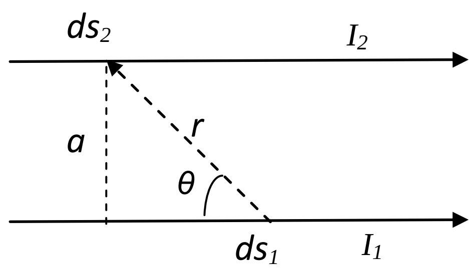

Fig.4. The charge dq_1 is located on the segment ds_1, while the charge dq_2 is on the segment ds_2.

Since the conductors are neutral, the electrostatic fields of the positive and negative charges cancel each other. Additionally, the accelerations of all the charges are zero, so in Weber's formula (2) the term depending on the accelerations disappears. Thus the simplified formulafor the force between elementary charges dq_1 and dq_2 is:

$$dF_W = \frac{dq_1 dq_2}{4\pi\varepsilon_0 r^2}\left[\frac{u^2}{c^2} - \frac{3}{2}\frac{\dot{r}^2}{c^2}\right] \quad (4)$$

Taking into account that $\dot{r}$is the projection of the relative velocity $\vec{u}$ onto the r direction, $\dot{r}^2 = (u\,cos\theta)^2 = u^2\frac{r^2-a^2}{r^2} = u^2\left(1 - \frac{a^2}{r^2}\right)$ and the formula becomes

$$dF_W = \frac{dq_1 dq_2}{4\pi\varepsilon_0}\frac{1}{r^2}\frac{u^2}{c^2}\left(\frac{3a^2}{2r^2} - \frac{1}{2}\right) \quad (4')$$

It is necessary to sum up such forces for each combination of charge sign on segment ds_1 with each combination of the same on segment ds_2 - altogether four such combinations are possible. Each combination has different relative velocity u and the sign of $dq_1 dq_2$, which is positive when the charges have the same sign and negative otherwise. Finally we obtain

$$dF_{4W} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2c^2}\left(\frac{3a^2}{2r^2} - \frac{1}{2}\right)U|dq_1| \cdot |dq_2|$$

where $U = (v_2^+ - v_1^+)^2 - (v_2^- - v_1^+)^2 - (v_2^+ - v_1^-)^2 + (v_2^- - v_1^-)^2$.

After squaring, all squares of the velocities cancel out and remain
$U = -2v_2^+v_1^+ + 2v_2^-v_1^+ + 2v_2^+v_1^- - 2v_2^-v_1^- = -2(v_2^+ - v_2^-)(v_1^+ - v_1^-)$.

Using (3) and (3'), we obtain

$$U|dq_1| \cdot |dq_2| = -2I_1I_2ds_1ds_2$$

For each elementary charge dq_1 we take the projection of the force dF onto the direction perpendicular to the currents and calculate the integral sum of all these elementary forces. Then the charges of the bottom wire repel the segment ds_2 of the upper one with the following resulting force R_w, perpendicular to the currents

$$R_W = -\frac{2I_1I_2ds_2}{4\pi\varepsilon_0c^2}\int_{-\infty}^{\infty}\frac{ds_1}{r^2}\left(\frac{3a^2}{2r^2} - \frac{1}{2}\right)sin\theta \quad (5)$$

The negative sign means that the force is attractive when the currents havethe same directions. Substitution $\sin\theta = a/r$ and $r = \sqrt{s_1^2 + a^2}$ in (5) and integration with respect to s_1 yield

$$R_W = -\frac{2I_1I_2ds_2}{4\pi\varepsilon_0c^2}\frac{1}{a} = -\frac{\mu_0I_1I_2ds_2}{2\pi a}$$

where we have used the known equality for the vacuum permeability $\mu_0 = \frac{1}{\varepsilon_0c^2}$.

The last expression coincides with Ampère's force law [15]. It is important to emphasize that Weber's assumption about moving positive charges has no influence on the resulting Weber force for moving conductors. The latter remains equal to Ampère's force measured for conductors at rest.

4. Explanation of magnetic field paradoxes on the basis of Weber's force

In section 1 we described the paradox in which the neutral magnet exerted no force upon the charge at rest (Fig.1), whereas the Lorentz force is exerted on the same charge in the moving observer's frame. On the other hand, the uniform movement of the observer cannot affect the Weber force, which depends only on the relative velocities.

The paradox shown in Fig.2 demonstrates the Lorentz force acting upon the charges in a moving conductor. This force turns into emf of induction if viewed from the reference frame attached to the conductor. Again, according to Weber, there is no contradiction, because his force is the same irrespective of the movement of the observer. In such <u>built-in relativity</u> the role of an observer is reduced to zero, so there is no need to invent laws meeting this requiriment.

Let us consider Faraday's paradox (see Fig.3). In order to analyse the influence of the magnet rotation on the force exerted upon an electron moving in its field, we generalize Assis's derivation ([11], page 172) of Weber's force in a stationary solenoid for the case when the solenoid rotates around its axis (Fig.5). There are positive and negative charges dq_+ and dq_- on an area element dz dl of the solenoid surface. The element is neutral, so $dq_-=-dq_+$. Denoting by $\vec{v}_+$ and $\vec{v}_-$velocities of the elementary charges, we obtain the relation $dq(\vec{v}_+ - \vec{v}_-) = \vec{I}\, n\, dz\, dl$, where n is the number of turns per unit length of the solenoid, I is the current in each turn, so that $\vec{I}\, n\, dz$ is the current in a solenoid ring of width dz. As in Section 3, we allow the solenoid to rotate about its axis, so we do not require the speed $\vec{v}_+$ to be zero.

Let us calculate the elementary forces exerted on an electron by the charges dq_+ and dq_-. The electron is placed at point (0,0,0) and flies with velocity $\vec{v}_e$. Without loss of generality we can choose the direction of the axis Ox so that $\vec{v}_e=(v_x, 0, v_z)$.

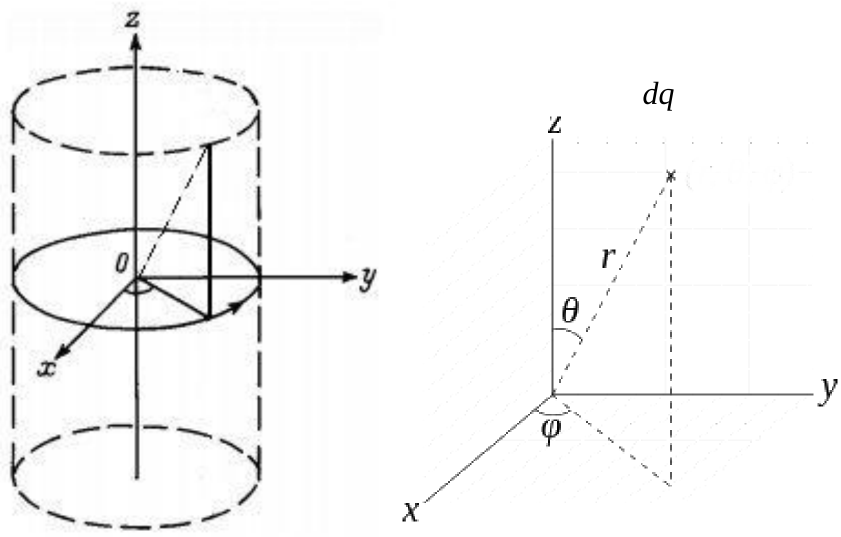

Fig.5. The coordinate system used for calculation of elementary Weber's forces and their projection on the Oy axis.

According to Weber's formula (2) the elementary forces on the electron by dq_+ and dq_- are as follows:

$$\mathrm{dF}_+ = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}\; dq_+}{\rho^2+z^2}\left[1+\frac{\vec{u}_+^2}{c^2}-\frac{3}{2}\frac{(\hat{r}\cdot\vec{u}_+)^2}{c^2}+\frac{\vec{r}\cdot\vec{a}_+}{c^2}\right] \qquad (6)$$

$$\mathrm{dF}_- = \frac{1}{4\pi\varepsilon_0}\frac{\mathrm{e}\; dq_-}{\rho^2+z^2}\left[1+\frac{\vec{u}_-^2}{c^2}-\frac{3}{2}\frac{(\hat{r}\cdot\vec{u}_-)^2}{c^2}+\frac{\vec{r}\cdot\vec{a}_-}{c^2}\right] \qquad (7)$$

where $\vec{u}_+ = \vec{v}_+ - \vec{v}_e$ is the velocity of the charge dq_+ relative to the electron velocity, and $\hat{r}$ is the unit vector in the $\vec{r}$ direction.

The speeds of the charges are directed tangentially to the cylinder, hence perpendicular to $\vec{r}$, therefore $\hat{r}\cdot\vec{v}_+ = 0$. After summing up (6) and (7) to get $\mathrm{dF} = \mathrm{dF}_+ + \mathrm{dF}_-$, the terms $\hat{r}\cdot\vec{v}_e$ will appear in the sum with opposite signs and will cancel out. As a result we obtain

$$\mathrm{dF} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{\mathrm{e}\;|dq|}{\rho^2+z^2}\left(\vec{u}_+^2 - \vec{u}_-^2 + \vec{r}\cdot\vec{a}_+ - \vec{r}\cdot\vec{a}_-\right). \qquad (8)$$

Next $\vec{u}_+^2 = (\vec{v}_+ - \vec{v}_e)^2 = \vec{v}_+^2 - 2(\vec{v}_+\cdot\vec{v}_e) + \vec{v}_e^2$. The last term is cancelled with the same for $\vec{u}_-^2$, hence $\vec{u}_+^2 - \vec{u}_-^2 = \vec{v}_+^2 - \vec{v}_-^2 - 2(\vec{v}_+ - \vec{v}_-)\cdot\vec{v}_e$.

The centripetal acceleration $\vec{a}_+$ is directed to the axis of the solenoid and equals $\vec{v}_+^2/\rho$. Let us denote by θ the angle between $\vec{a}_+$ and $\vec{r}$, then cosθ=ρ/r and

$$\vec{r}\cdot\vec{a}_{+} = r\frac{\vec{v}_{+}^{2}}{\rho}\frac{\rho}{r} = \vec{v}_{+}^{2}.$$

$$\mathrm{dF} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{\mathrm{e}\,|dq|}{\rho^2+z^2}\left[2(\vec{v}_{+}^{2}-\vec{v}_{-}^{2}) - 2(\vec{v}_{+}-\vec{v}_{-})\cdot\vec{v}_{e}\right]. \qquad (9)$$

The force dF is directed at an angle θ to the plane xOy (see Fig.5b). To calculate the projection of dF onto this plane we multiply dF by cosθ=ρ/r, and also multiply by sinφ to obtain the y-component of the force:

$$\mathrm{dF}_{y} = \frac{\mathrm{e}\rho}{4\pi\varepsilon_0 c^2}\frac{|dq|sin\varphi}{(\rho^2+\mathrm{z}^2)^{3/2}}[2(\vec{v}_{+}^{2}-\vec{v}_{-}^{2})|dq| - 2|dq|(\vec{v}_{+}-\vec{v}_{-})\cdot\vec{v}_{e}]$$

Now we use $\vec{v}_e$=(v_x, 0, v_z), $\vec{I}$=(-*I*sinφ, *I*cosφ, 0), so that the second term becomes
$-2(\vec{v}_e\vec{I})\, n \,\mathrm{dz}\,\rho\,\mathrm{d}\varphi = 2v_x I\, n\,\mathrm{dz}\,\rho\,\sin\varphi\,\mathrm{d}\varphi$.

To get F_y we integrate dF_y with respect to φ from 0 to 2π and with respect to z from -∞ to ∞ which yields

$$\mathrm{F}_{y} = \mu_0 e\, v_x I\, n \qquad (10)$$

where we have used the known equality for the vacuum permeability $\mu_0 = \frac{1}{\varepsilon_0 c^2}$.

It appears from (10) that Weber's force does not include $\vec{v}_{+}$, i.e. rotation of the magnet has no effect on the force exerted upon the moving electron. The question whether the magnetic field rotates with the magnet or not is meaningless because the magnetic field is a mathematical concept to which the property of rotation is not applicable.

Since for a solenoid the relation B = $\mu_0\, n\, I$ holds, we may write

$$\mathrm{F}_{y} = \mathrm{e}\, v_x B$$

This means that Weber's force on the electron moving in a solenoid coincides with the Lorentz force in stationary case. However it remains unknown whether rotation of the solenoid affects the Lorentz force or not.

5. *Weber's force and mass variation*

Albert Einstein used the concepts of longitudinal and transverse masses in his 1905 paper on electrodynamics (see [16]). Let m_0 be the object mass at rest and β=*v*/c, where v is the object velocity, then the transverse mass $m_T = \frac{m_0}{\sqrt{1-\beta^2}}$ perpendicular to the direction of motion, whereas the longitudinal mass $m_L = \frac{m_0}{(1-\beta^2)^{3/2}}$ parallel to the same direction.

Since then many experiments have been carried out to verify the relativistic formula

$$m_T = \frac{m_0}{\sqrt{1-\beta^2}} \qquad (11)$$

Among them were Kaufmann [17], Bucherer [18], Neumann [19], and others. In discussing the results of these experiments, only different formulas for mass-velocity variation were compared. It is believed that the results of experiments have confirmed the special relativity relation (11).

Weber died in 1891, fourteen years before the relativity theory announcement. Weber's force formula (2) for moving charges does not require a mass-velocity variation in order to explain the experimental results. If Weber were alive, he would have proved it, but the first proof was published by V.Bush [13] only in 1926. Bush showed (p148) that the integral Weber force acting on an electron moving along the parallel plates of a capacitor is perpendicular to the electron velocity and equals

$$F_W = Ee\left(1 + \frac{\beta^2}{2}\right) \qquad (12)$$

where ***E*** is the electric field inside the capacitor. In Bucherer's [18] and Neumann's [19] experiments relativistic electrons were moving between the capacitor plates that worked as a velocity selector by means of perpendicular magnetic field **B**. After leaving the capacitor the electrons continued moving in the same magnetic field but in a circular trajectory, whose radius r was measured. V.Bush compared the following two interpretations of the experiments:

<u>Interpretation of the experiments from the standpoint of Lorentz</u>: inside the capacitor electrical and magnetic forces are balanced, therefore

$$Ee = e\, v\, B. \qquad (13)$$

Outside the capacitor the centripetal force is supplied by the Lorentz force

$$\frac{m\ v^2}{r} = e\ v\ B \qquad (14)$$

therefore v=E/B and

$$\frac{e}{m} = \frac{E}{rB^2}. \qquad (15)$$

The right side of (15) contains only measured values. It is not a constant, which means that the left side is also variable. Using (11) in (15) yields

$$\frac{e}{m_0/\sqrt{1-\beta^2}} = \frac{E}{rB^2}. \qquad (16)$$

The deviations of the calculated values e/m_0 do not exceed 1-2% of the average value. This may be considered as good agreement compared to other mass-velocity dependencies.

Interpretation of the experiments from the standpoint of Weber: based on (12) the value of electric force on the electron must be multiplied by $(1+\beta^2/2)$:

$$E\left(1+\frac{\beta^2}{2}\right)e = e\ v\ B, \qquad (13')$$

Weber assumed a constant mass m_0. As we noted above, Weber's force on a charge moving in magnetic field coincides with the Lorentz force. Hence, there are no other changes in formulas (13) - (15). As the result, instead of (15) we obtain

$$\frac{e}{m_0} = \frac{E\left(1+\beta^2/2\right)}{rB^2}. \qquad (15')$$

or

$$\frac{e}{m_0\left(1+\beta^2/2\right)} = \frac{E}{rB^2}. \qquad (16')$$

A series expansion of $1/\sqrt{1-\beta^2}$ to the terms of the order β^3 yields

$$\frac{1}{\sqrt{1-\beta^2}} \approx 1+\beta^2/2.$$

Comparing (16) and (16 ') we ascertain that up to terms of order β^3 both formulas coincide.

It should be noted that the velocity of the electrons in the experiments were not measured, but calculated on the basis of a model instead. To account for this fact we will use the following two symbols for v/c: β_L= E/cB for Lorenz's model and $\beta_W\approx$ E/cB[1+(E/cB)^2] for Weber's model up to terms of order (E/cB)^3 (see Assis [11], p.175). Then

$$\frac{1}{\sqrt{1-\beta_L^2}} \approx 1+\frac{\beta_L^2}{2} = 1+\left(\frac{E}{cB}\right)^2/2$$

$$1+\frac{\beta_W^2}{2} = 1+\left[\frac{E}{cB}\left(1+\left(\frac{E}{cB}\right)^2\right)\right]^2/2 \approx 1+\left(\frac{E}{cB}\right)^2/2$$

This means that the factors of m_0 in (16) and (16′), expressed only through the measured values, coincide with the terms of the order (E/cB)^3 for the two interpretations of the experiments. So the electron mass remains constant in Weber's interpretation of Bucherer's [18] and Neumann's [19] experiments.This meets the requirement of Occam's razor.

Discussion

At first glance it appears that both interpretations are equally good; however Lorentz's interpretation violates the conditions for validity of an electrostatic formula when applied to the relativistic electrons. Let us begin with Coulomb's electrostatic force, describing the interaction between two electrical charges at rest [9]: "Coulomb's law is fully accurate only when the objects are stationary". Now, the magnitude of electric field E is defined as follows [20]: "The electric field E at a given point is defined as the (vectorial) force F that would be exerted on a stationary test particle of unit charge by electromagnetic forces (i.e. the Lorentz force). A particle of charge q would be subject to a force F=q E."

Coulomb's law cannot guarantee that the force on the test charge does not vary with its speed. Thus for a moving test charge we will get different forces F and hence different magnitudes of the electric field. Hendrik Lorentz (see [21]) was the first who began to use freely the expression F = q E for electric force calculations in 1892.

The expression F = q E was also used in discussions of the experiments carried out by Kaufmann, Bucherer, Neumann starting from 1905. A quote from Neumann [19], p.531 is as follows: "so ist die elektrostatische auf das Elektronausgeübte Kraft e•E, die elektrodynamische e•H•u." (translation: so the electrostatic force exerted on the electron is *eE*, the electrodynamic one is *eHu*). Neumann well understood that *eE* is an electrostatic force, nevertheless he used it for the relativistic electrons in (13), thus infringing the limits of the laws of electrostatics.

Conclusions

1. Weber's force law leads to Ampere's force law, to Faraday's induction law, to the Lorentz force on a charge flying in a solenoid. Weber's force for elementary interactions between charges is directed along the line connecting these charges, so that Newton's third law is not violated.

2. In the experiments with deviation of relativistic electrons by a transverse electric field, Weber's force predicts up to terms of (v/c)^3 the same relation between the measured values as the variable mass of special relativity, i.e. Weber's force eliminates the necessity of introducing the variable electron mass.

The importance of Weber's approach is that it rids physics of two redundant models without reducing in its predictive ability. In addition, the approach explains the previously inexplicable Faraday paradox and supplies simple explanations for other paradoxes of electromagnetism. All this is due to the fact that the relativity is embedded in the very basis of the interaction of charges.

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